

: kind of a Pointless* Talk

It's not about tacit programming!



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Terminology

Value-level Terms (Expressions)

Types

Kinds

Expressions

23

“Hello World!”

23 + 42

True && False

Types

> 23 :: Int

> "Hello World!" :: String

> 23 + 42 :: Int (assuming + :: Int → Int → Int)

> True && False :: Bool (assuming && :: Bool → Bool → Bool)

Kinds

Types of Types

Kind * aka Type

Types of kind * can have values.

Kinds of Simple Types

> `Int :: *`

> `String :: *`

> `Int → Int → Int :: *`

Kinds of Too Simple Types

> `Int :: *`

> `String :: *`

> `Int → Int → Int :: *`

Custom Data Types

Still :: *

```
data Bool = True | False
```

```
data List'of'Ints = Nil  
                  | Cons Int List'of'Ints
```

```
empty          = Nil
```

```
a'few'numbers = Cons 1 (Cons 2 (Cons 3 Nil))
```

Polymorphic Data Types

Types Abstracting over other Types

```
data List a = Nil  
           | Cons a (List a)
```

```
data Maybe a = Nothing  
            | Just a
```

```
data Either a b = Left a  
               | Right b
```

Kinds of Polymorphic Data Types

aka Type Constructors

> `List` :: * → *

> `Maybe` :: * → *

> `Either` :: * → * → *

Higher-kinded Types

Types Abstracting over Types Abstracting over Types

Higher-kinded Types

Types Abstracting over (Types Abstracting over Types)

Higher-kinded Types

```
data Container m a = Contain (m a)
```

```
> :kind Container
```

Higher-kinded Types

```
data Container m a = Contain (m a)
```

```
> :kind Container
```

```
> Container :: (* -> *) -> * -> *
```

example

```
data Container m a = Contain (m a)
```

```
list'of'int = Contain (Cons 1 Nil)
```

```
> :type list'of'int
```

```
> list'of'int :: Container List Int
```


Grammar of Kinds

Kind $k, \lambda = *$
 $| k \rightarrow \lambda$

Kind Polymorphism

Exposing lies (mostly mine)

```
data Container m a = Contain (m a)
```

```
> :kind Container
```

Kind Polymorphism

Exposing lies (mostly mine)

```
data Container m a = Contain (m a)
```

```
> :kind Container
```

```
> Container :: (k → *) → k → *
```

Custom Kinds

For more kind-level goodness.

```
data Response i = R String
```

```
data Valid
```

```
data Unknown
```

```
data Response i = R String
```

```
data Valid
```

```
data Unknown
```

```
validate :: Response Unknown → Maybe (Response Valid)
```

```
data Response i = R String
```

```
data Valid
```

```
data Unknown
```

```
validate :: Response Unknown → Maybe (Response Valid)
```

```
derp :: Response Bool
```

```
data Response i = R String
```

```
data Valid
```

```
data Unknown
```



```
Response Unknown → Maybe (Response Valid)
```

```
derp :: Response Bool
```


Back to the drawing board

Let's engage those galaxy brains




```
data Response (i :: Response' I) = R String
```

```
kind Response' I
```

```
data Valid :: Response' I
```

```
data Unknown :: Response' I
```

```
validate :: Response Unknown → Maybe (Response Valid)
```

```
data Response (i :: Response'I) = R String
```

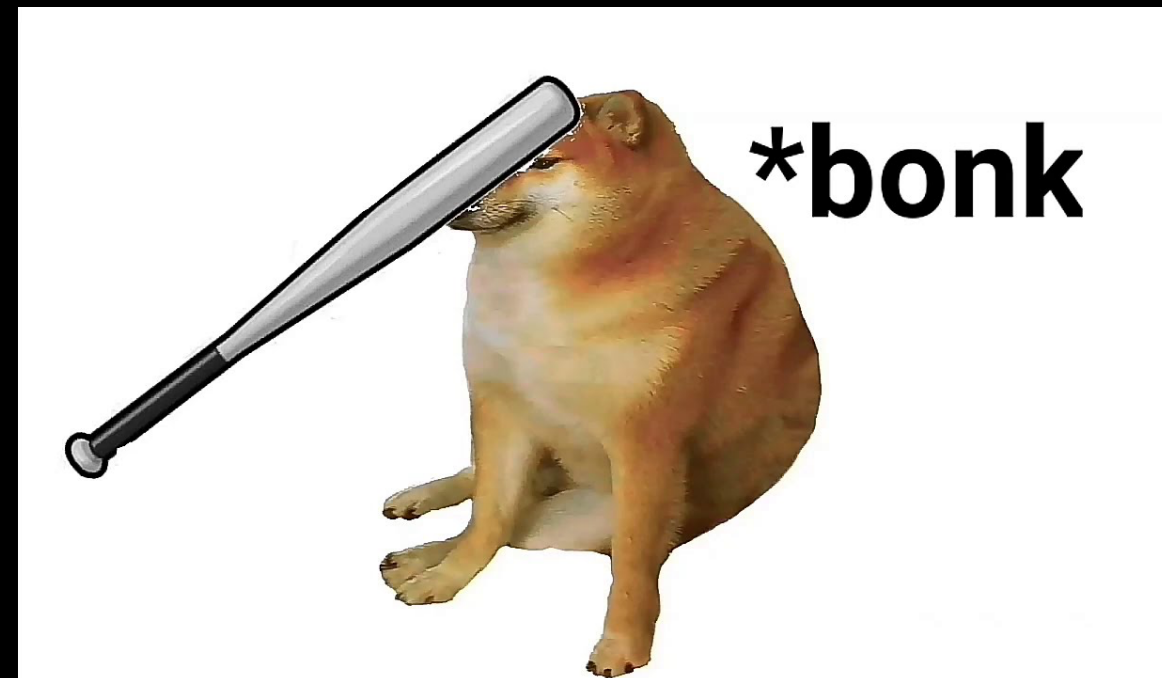
```
kind Response'I
```

```
data Valid :: Response'I
```

```
data Unknown :: Response'I
```

```
validate :: Response Unknown → Maybe (Response Valid)
```

```
derp :: Response Bool
```



What about Haskell?

Haskell unifies Types and Kinds.

Extensions like `DataKinds` promote types into kinds and data constructors into types.

Haskell does not provide a facility showed in the previous slides. (It does not need it though.)

Values with Types of Custom Kinds

The what now?

The Fantasy Land part of the Talk.

Kind * aka Type

Types of kind * can have values.

Kind * aka Type

Only types of kind * can have values.

Kind * aka Type

~~Only types of kind * can have values.~~

```
kind Foo'Kind
```

```
type Foo'Type :: Foo'Kind
```

```
data Foo'Type = Foo'Val
```



```
kind Foo'Kind
```

```
type Foo'Type :: Foo'Kind
```

```
data Foo'Type = Foo'Val
```

```
some'foo = Foo'Val
```

```
kind Foo'Kind
```

```
type Foo'Type :: Foo'Kind
```

```
data Foo'Type = Foo'Val
```

```
some'foo = Foo'Val
```

```
> :type some'foo
```

```
kind Foo'Kind
```

```
type Foo'Type :: Foo'Kind
```

```
data Foo'Type = Foo'Val
```

```
some'foo = Foo'Val
```

```
> :type some'foo
```

```
> some'foo :: Foo'Type
```

```
kind Foo'Kind
```

```
type Foo'Type :: Foo'Kind
```

```
data Foo'Type = Foo'Val
```

```
id'foo ::  $\forall (a :: \text{Foo'Kind}) . a \rightarrow a$ 
```

```
id'foo x = x
```

```
kind Foo'Kind
```

```
type Foo'Type :: Foo'Kind
```

```
data Foo'Type = Foo'Val
```

```
id'foo ::  $\forall (a :: \text{Foo'Kind}) . a \rightarrow a$ 
```

```
id'foo x = x
```

```
> id'foo Foo'Val
```

```
> Foo'Val
```

```
> id'foo True
```

```
> ERROR!
```

```
kind Foo'Kind
```

```
type Foo'Type :: Foo'Kind
```

```
data Foo'Type = Foo'Val
```

```
id'foo ::  $\forall (a :: \text{Foo'Kind}) . a \rightarrow a$ 
```

```
id'foo x = x
```

```
> id'foo Foo'Val
```

```
> Foo'Val
```

```
> id'foo True
```

```
> ERROR!
```



So what about our ordinary `identity` function?

```
id x = x
```

```
> id Foo'Val
```

```
> ERROR!
```

Let's fix that

The real identity™

$\text{id} :: a \rightarrow a$
 $\text{id } x = x$

Let's fix that

The real identity™

$id :: \forall (a :: *) . a \rightarrow a$
 $id\ x = x$

Let's fix that

The real identity™

$\text{id} :: \forall (a :: *) . a \rightarrow a$
 $\text{id } x = x$

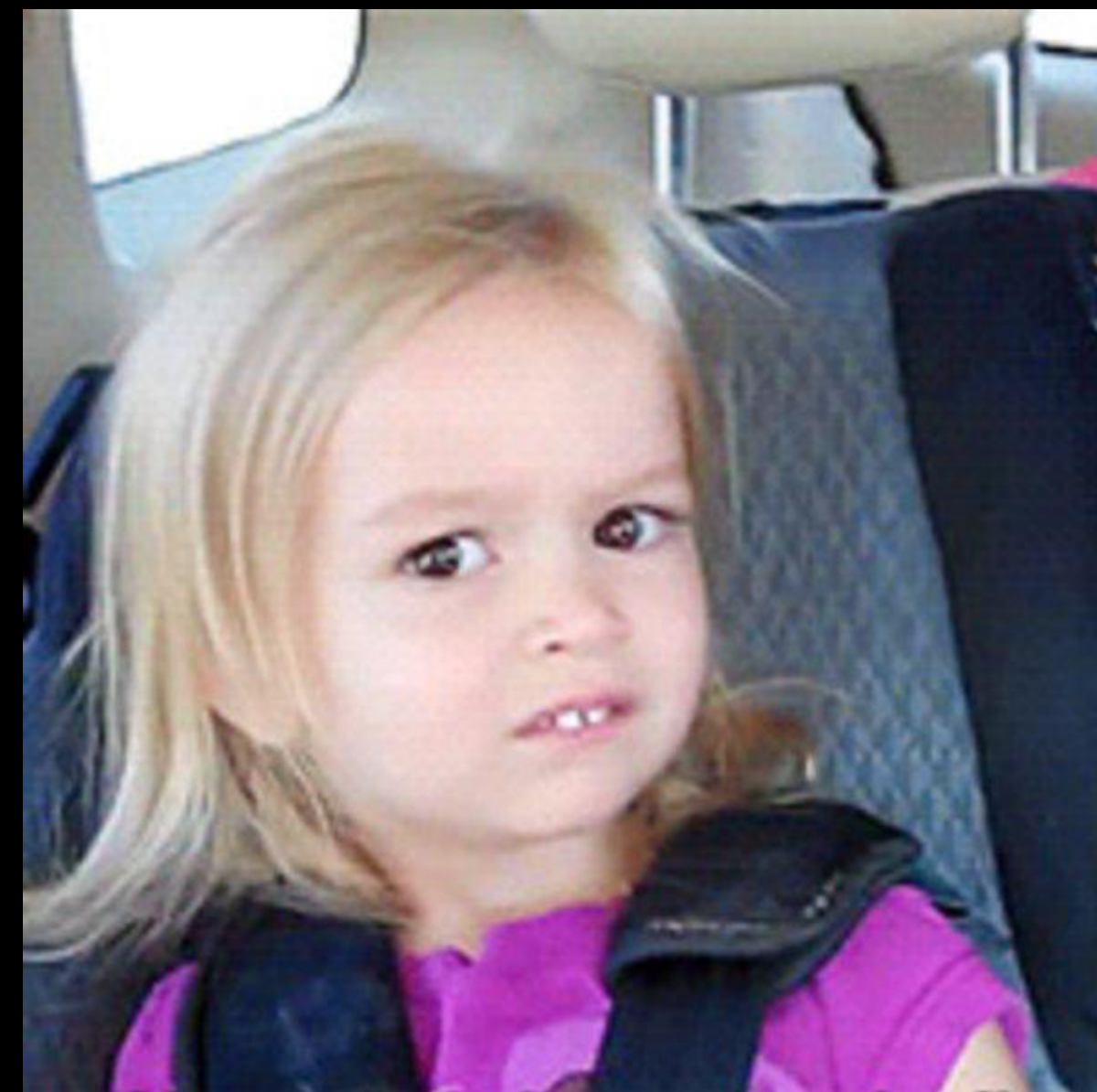
$\text{real'id} :: \forall (a :: k) . a \rightarrow a$
 $\text{real'id } x = x$

Let's fix that

The real identity™

$\text{id} :: \forall (a :: *) . a \rightarrow a$
 $\text{id } x = x$

$\text{real'id} :: \forall (a :: k) . a \rightarrow a$
 $\text{real'id } x = x$



```
real'id :: ∀ (a :: k) . a → a  
real'id x = x
```

```
> real'id Foo'Val  
> Foo'Val
```

```
> real'id True  
> True
```

Uh, oh

```
data Broken (x :: k) = Break x
```

Uh, oh

```
data Broken (x :: k) = Break x
```

```
broken :: Broken Maybe
```

```
broken = Break ???
```

Uh, oh

```
data Broken (x :: k) = Break x
```

```
broken :: Broken Maybe  
broken = Break ???
```



No wait, we can fix this!

But how?

Maybe with sub-kinding?

Maybe with set-theoretical kind polymorphism?



Sub-kinding?

$A \leq B$ means A is a sub-kind of B

```
kind Foo'Kind ≤ *
```

```
type Foo'Type :: Foo'Kind
```

```
data Foo'Type = Foo'Val
```

```
real'id :: ∀ (a ≤ *) . a → a
```

Set-theoretical kind polymorphism?

* | **Foo'Kind** is a union of those two kinds.

`real'id :: ∀ (a :: * | Foo'Kind) . a → a`

`real'id x = x`

That is a Different Talk though.

Kind * aka Type

Only types of kind * can have values.

Kind * aka Type

~~Only types of kind * can have values.~~

Kind # in Haskell

Kind for unlifted types.

Also see levity-polymorphism in GHC.

Remember to be kind!

Resources

- https://www.parsonsmatt.org/2017/04/26/basic_type_level_programming_in_haskell.html
- https://downloads.haskell.org/~ghc/7.8.4/docs/html/users_guide/kind-polymorphism.html
- <https://wiki.haskell.org/Kind>